GENERALIZED HANKEL-TYPE INTEGRAL TRANSFORM ON $L_{\nu,r}$-SPACES

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Dedicated to 50th anniversary of Virginia Kiryakova

Abstract

The paper is devoted to the study of the integral transform

$$(H_{\eta;\sigma,\omega;a,b,\lambda}f)(x) = x^{\sigma}\int_0^{\infty} J_{\eta}(\lambda x^a t^b)\, t^{\omega} f(t)\, dt \quad (x > 0),$$

with $\eta \in \mathbf{C}$, $Re(\eta) > -1$, $\sigma \in \mathbf{C}$, $\omega \in \mathbf{C}$, $\lambda > 0$, $a > 0$ and $b > 0$, containing the Bessel function of the first kind $J_{\eta}(z)$ in the kernel on the space $L_{\nu,r}$ ($\nu \in \mathbf{R} = (-\infty,\infty)$, $\mathbf{1} \leq \mathbf{r} \leq \infty$) of Lebesgue measurable functions f on $\mathbf{R}_{+} = (\mathbf{0},\infty)$ such that

$$\int_0^{\infty} |t^{\nu} f(t)|^r \frac{dt}{t} < \infty \quad (1 \leq r < \infty,\, \nu \in \mathbf{R}),$$

$$\underset{t>0}{ess\ sup} \left[t^{\nu}|f(t)|\right] < \infty \quad (r = \infty).$$

The mapping properties such as the boundedness, the representation and the range of the transform $H_{\eta;\sigma,\omega;a,b,\lambda}$ are proved, and the inversion formulae are established. The special case is considered.

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